

# *The Effect of the Size and Position of the Crack on the Normalized Stress Intensity Factor*

*Mostefa BENDOUBA<sup>1</sup>, Abdelkader DJEBLI<sup>1</sup>, Abdelghani BALTACH<sup>1,2</sup>, Ali BENHAMENA<sup>1</sup>, Amel BOUKHLIF<sup>1</sup> and Abdelkrim AID<sup>1</sup>*

<sup>1</sup>Laboratory of Quantum Physics of Matter and Mathematical Modeling (LPQ3M), University Mostapha Stambouli-Mascara, Mascara 29000, Algeria.

<sup>2</sup>Department of Mechanical Engineering, University of Tiaret, Tiaret, Algeria

\*Corresponding author Email: [baltachabdelghani@yahoo.fr](mailto:baltachabdelghani@yahoo.fr),

---

## Article Info

### Article history:

Received 12 March 2020

Revised 26 April 2020

Accepted 15 May 2020

### Keywords:

Finite element method  
Normalized stress intensity factor  
Nodal displacement extrapolation method  
Energy method  
Rice integral

---

## ABSTRACT

In this work, finite element method was used to determine the normalized stress intensity factors for different configurations. For this, a 2-D numerical analysis with elastic behavior was undertaken in pure I mode. This simulation was carried out using a numerical calculation code. On the basis of the numerical results obtained from the different models treated, there is a good correlation between the nodal displacement extrapolation method (DEM) and the energy method based on the Rice integral (J) to evaluate the normalized stress intensity factors and this for different crack lengths. For each configuration, the increase in the crack size causes an amplification of normalized intensity stresses factors.

---

## I. Introduction

Most structural engineering components fail under the action of non-static loading. The cyclical stresses resulting from this type of loading cause physical degradation of the materials involved. Over time, the accumulated damage can cause the appearance and growth of cracks that end up rendering structures or components unusable. This process is called "fatigue", since the alternating stresses gradually decrease the mechanical resistance of the material. Catastrophic mechanical failures due to the unstable spread of cracks originating in stress concentrators have caused financial loss and caused deaths around the world. Understanding the crack origin process in a given discontinuity and its propagation to failure is of fundamental importance for the elaboration of inspection and maintenance plans in machinery and equipment in order to minimize the occurrence of such deleterious processes. The use of fracture mechanics in engineering projects has evolved a lot in recent years, mainly due to the use of numerical methods. These are used in the determination of fracture toughness parameters, in the analysis of stresses and / or strains in structures containing cracks and in the study of crack growth. Several authors [1–4] have been studied the fracture problems of mechanical components by means of experimentation and numerical simulation in order to assess the mechanical integrity, taking into account different crack shapes in various application. Some examples of materials that can be analyzed by the fracture mechanics are: all high-strength materials from the aerospace industry [5], high-strength and low-alloy steels, cold-deformed stainless steels and in piping systems of oil [6]. In the same context, a thorough examination of this subject can be found in Refs. [7–13]. To determinate the

fracture toughness parameters, there are several methods to calculate the stress intensity factor: those that use the correlation of the stress fields and / or displacement at the crack tip [1], hybrid methods [14], J integral [15-16], the approach of strain energy [17] and the virtual crack extension technique [18]. The present study aims to use the techniques that use the displacement field at the crack tip (Displacement Extrapolation Method: DEM) and (J Integral Method: JIM) to determine the normalized stress intensity factor for various geometry. The procedures were implemented in a commercial software for analysis by the finite element method; Abaqus 6.14. This software allows the creation of routines, through its own programming language. Thus, becoming a system suitable for the implementation and performance of analysis within the field of fracture mechanics. A good correlation was found between the FEM simulations and the literature results.

## II. Hollow cylinder cracked under internal pressure

### II.1. Geometrical model

To determine the Normalized stresses intensity factor, hollow cylinders with axial external crack is considered by Stefan-Dan [6] and CP. Andrasic [7]. Fig 1 presents the geometrical model and the load of the series of hollow cylinders pressurized with an internal pressure  $P = 1\text{MPa}$ .

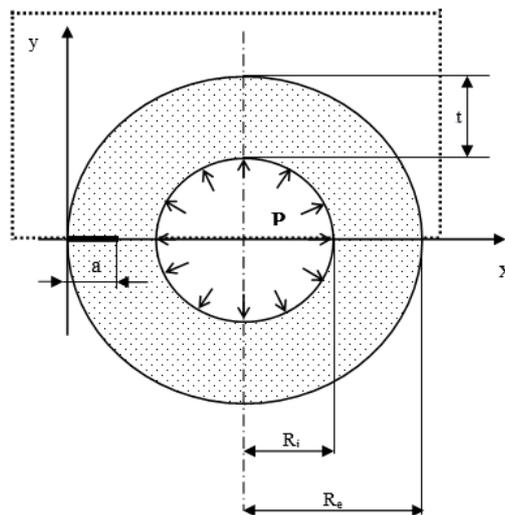


Figure 1. Geometrical model [6].

The dimensions characteristics of the cylinder are as follows:

- Internal radius:  $R_i = 20\text{ mm}$  ;
- External radius:  $R_e = 40\text{ mm}$  ;

### II.2. Mechanical properties

Material used in this study is an En 24 grade steel. However, the mechanical properties are given in Tab. 1.

Mechanical properties	En 24 grade steel
Yong modulus E (GPa)	210
Poisson coefficient $\nu$	0.3

### II.3. FE model and boundary conditions

For each crack length, a finite element analysis was performed in order to obtain the numerical values of the stress intensity factor. Then, a comparison of the numerical results with those of the literature. Thus, these

results are exploited to determinate the displacements of the surface of crack. Considering the symmetry of the geo-metrical model and the loading conditions along the axe X–X, only a half of the model was simulated in order to minimize the time of computation. Hollow cylinder was modeled with quadratic quadrilateral elements with 8 nodes of type CPS8R with functions of quadratic form. The elastic analysis is performed using these elements and has the advantage that the stress singularity at the crack tip can be incorporated in the the solution by moving the eight nodes to the quarter-point locations [8]. The mesh was refined at the point of crack, because the results obtained converge with an optimal time. The number of elements used is 1072 for the cracked structure. Figure 2 shows the EF model of the studied tube.

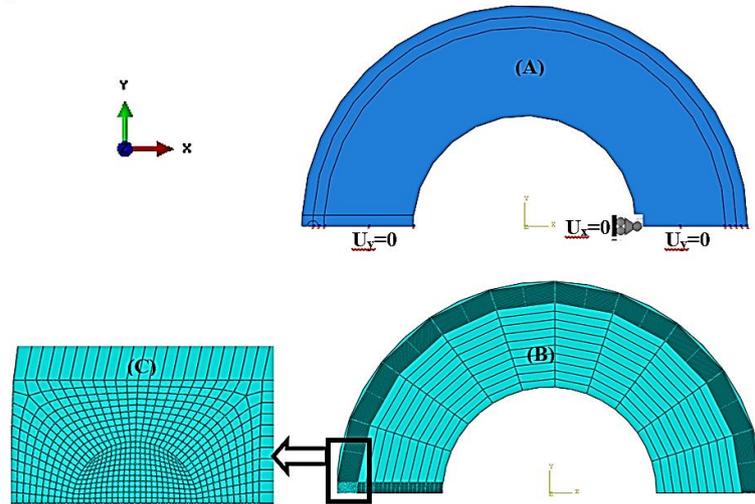


Figure 2. FE model of cracked Hollow cylinder: (a) bondoury conditions; (b) model and typical mesh; (c) mesh around the crack tip.

The boundary conditions and the mesh are represented in figure 2a and 2b successively. Then, the results obtained numerically are compared with those obtained by [9]. Normalized stress intensity factors are clacluated by the following relation:

$$Normalised\ SIF\ (K_{nor}) = \frac{K_I}{P\sqrt{\pi a}} \quad (1)$$

Where:

$P$ : internal pressure;

$a$ : crack lengths;

$K_I$ : stress intensity factors under pure mode I.

## II.4. Results and discussion

The analysis was done under pure mode I tension condition for various crack lengths ( $a$ ) ranging between  $a = 0.1 \times t$  and  $a = 0.9 \times t$ , where ( $t$ ) is the Hollow cylinder width. Figure 3 shows the evolution of the normalized stresses intensity factors  $K_{nor}$  with respect to the crack length characterized by the ratio " $a/t$ ".

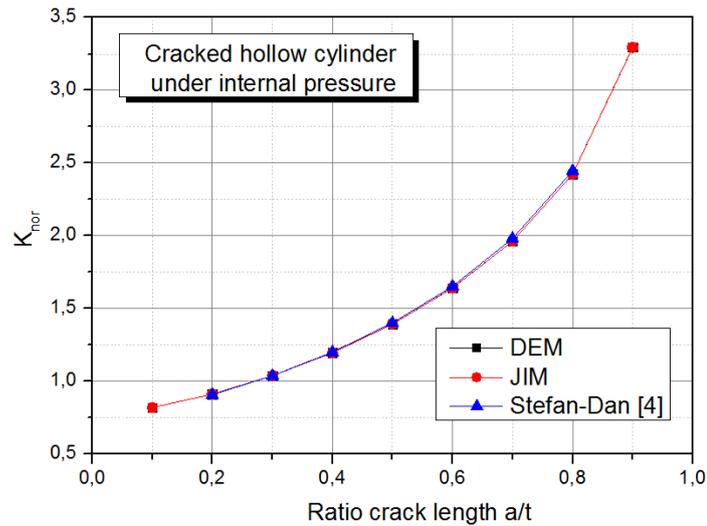


Figure 3. Variation of  $K_{nor}$  versus the crack length.

It is clear from fig. 3 that the  $K_{nor}$  value increases when the  $a/t$  ratio increases from 0.1 to 0.9. Another observation drawn from the results illustrated by this figure is that the curves are identical, which indicates that our finite element model is valid for the study of cylinders.

In effect, Figure 4 shows the ISO values representing the distribution of the stresses of von Mises for a length of crack  $a = 18\text{mm}$ .

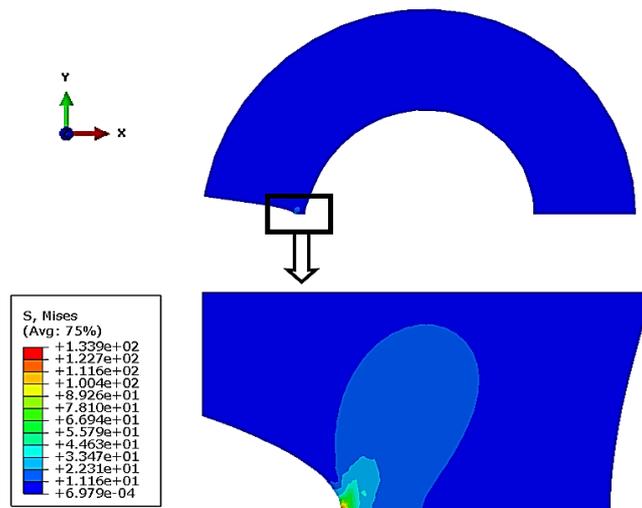


Figure 4. ISO- image of the stress distributions in the vicinity of the crack length  $a=18\text{mm}$

From this figure, the stress intensity is maximum at the crack tip.

It is well known that, a linear relation [10] links the stress intensity factor to the crack opening displacement (COD). Therefore, in order to check the relevance of the results obtained, the vertical displacements of the nodes on the crack lip obtained from the analysis by finite elements for all the crack lengths studied are identified.

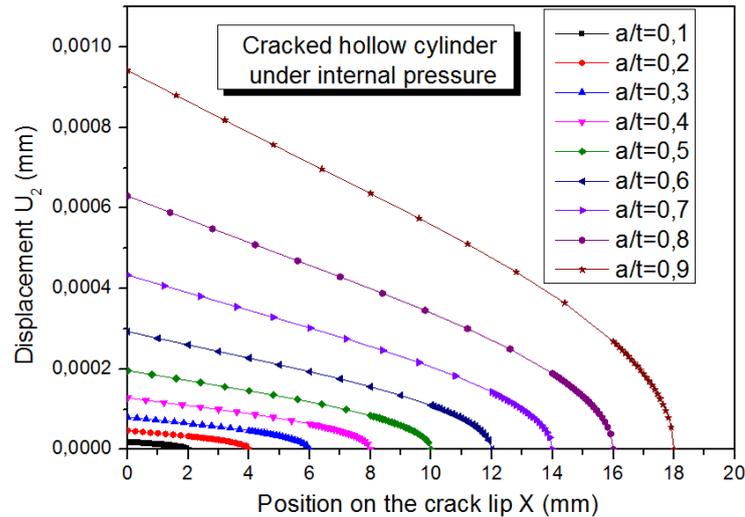


Figure 5. Variation of displacements of the crack surface for different lengths of cracks

Values of COD (Crack Opening Displacement) for all the lengths of crack are shown in Figure 5. Analysis of this figure shows that the size of the crack ( $a / t$ ) affects significantly on the COD values, i.e. the COD value increases with the increase in the crack length. A similar behavior is observed for all the crack lengths. Consequently, this confirms the existence of a direct correlation relation between the crack opening and the stress intensity factors.

### III. Hollow cylinder cracked under external tension

#### III.1. Geometrical model and mechanical properties

Consider an elastic hollow cylinder cracked under external tension [11-12] with dimensions shown in fig. 6. The material is assumed to be isotropic with shear modulus  $G = 100 \text{ Gpa}$  and Poisson's ratio  $\nu = 0.3$ .

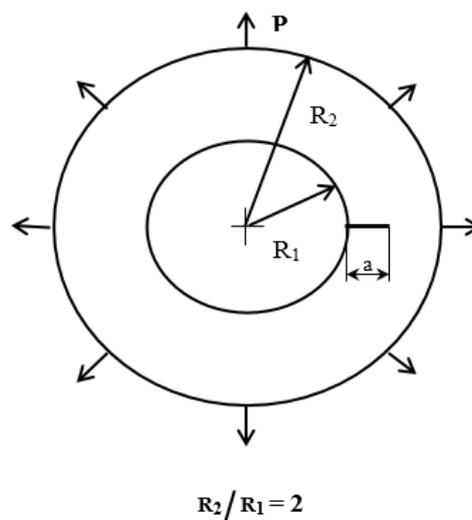


Figure 6. Geometrical model [11].

The dimensions characteristics of the cylinder are as follows:

- Internal radius:  $R_i = 20 \text{ mm}$  ;

- External radius:  $R_e = 40$  mm ;

### III.2. Results and discussion

In this study, a numerical model with the code (FEM Abaqus) was carried out for the numerical calculation of normalized intensity factors. Then, a comparison of the numerical results with those of the literature. The evolution of the normalized stresses intensity factors  $K_{nor}$  according to the length of crack characterized by the ratio ( $a/t$ ) is represented in figure 7.

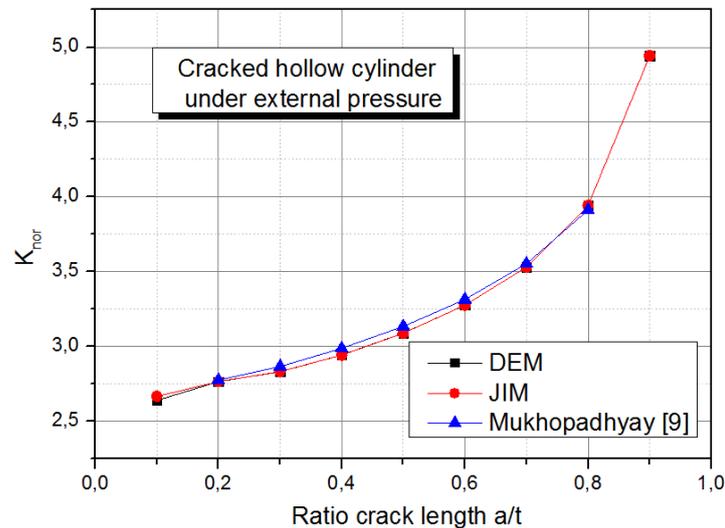


Figure 7. Variation of  $K_{nor}$  versus the crack length

This latter, illustrates the variation of  $K_{nor}$  according to the variation of the crack length ( $a/t$ ) for a cracked hollow cylinder under external tension. These results show a considerable effect on the evolution of this parameter  $K_{nor}$  as a function of the size of the crack ( $a/t$ ). Indeed, for a large range of ( $a/t$ ),  $K_{nor}$ 's value increases rapidly. In fact, normalized stress intensity factor  $K_{nor}$  increases exponentially for high ratios ( $a/t$ ) while the evolution of this parameter  $K_{nor}$  is almost remains linear for low ratios ( $a/t$ ). This variable behavior is probably related to the distribution of local stress due to the change in crack size. ie with the rigidity of the structure. These results clearly show the good agreement between the different techniques.

Figure 8 present the distribution of von-mises stresses in the cracked structure. The crack length is taken to be  $a=18$  mm.

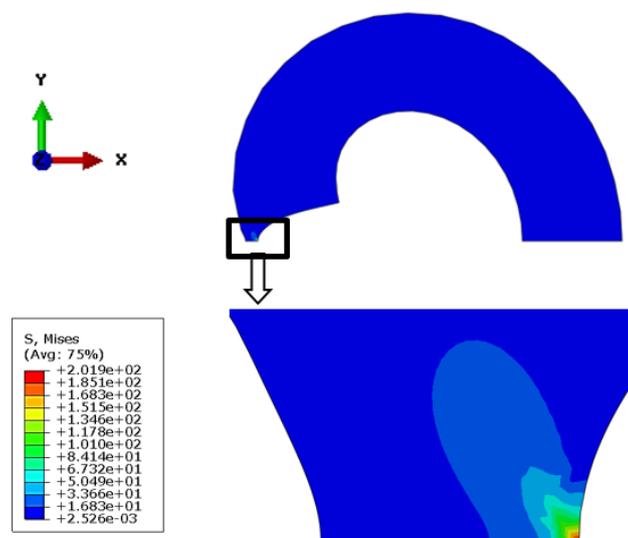


Figure 8. ISO- image of the stress distributions in the vicinity of the crack length  $a=18$ mm

From the figure above, the distribution of von-mises stresses in the vicinity of the crack shows that stress intensity is maximum at the crack tip (see the red intensity in fig. 8).

The effect of crack size on the value of COD (Crack Opening Displacement) is shown in the figure.9.

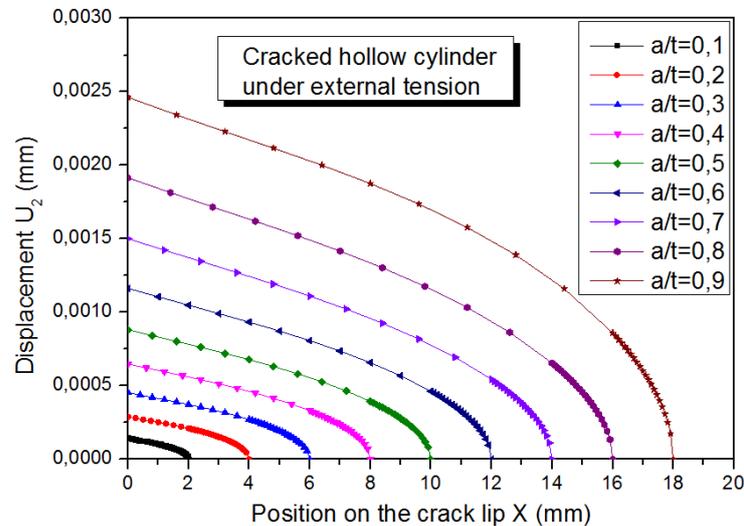


Figure 9. Variation of displacements of the crack surface for different lengths of cracks.

In the latter, the same tendency was observed for all configurations. Indeed, it can be seen from this figure that the values of COD increase with the increase of the length crack. This tendency is in good agreement with the results reported in the literature.

#### IV. Conclusion

The main objective of fracture mechanics is to predict the behavior of expected cracks found in all industrial components subjected to mechanical stresses. Then, the present work relates to the study of the behavior of cracked structures in failure of in two-dimensional linear elasticity by the finite element method in pure I-mode. The numerical results shown that:

- For a crack structure, an increase in the size of the crack causes to an increase in the normalized stress intensity factor and consequently the rupture of the mechanical component.
- Whatever the cracked geometric configuration, there is a good correlation between the nodal displacement extrapolation method (DEM) and the energy method based on the Rice integral (J) to evaluate the normalized stress intensity factor  $K_{nor}$ .
- The size of the crack has a significant effect on the COD values, that is to say the COD value increases with the increase in the crack length.

#### References

[1] A. Griffith., "The phenomena of rupture and flow in solids," *Philosophical Transactions of the Royal Society of London*, Series A, 221, pp. 163-198, 1920. DOI: <https://doi.org/10.1098/rsta.1921.0006>.

[2] G.R. Irwin., "Estimates of stress intensity and rivet force for a crack arrested by arivited stiffener. Discussion based on 'Analysis of stress and strains near the end of a crack traversing a plate,'" *Journal of Applied Mechanics*, vol. 24, pp. 361-364, 1957.

- [3] J.R. Rice., "A path independent integral and approximate analysis of strain concentrations by notches and cracks," *Journal of Applied Mechanics*, vol. 35, pp. 379-386, 1968. DOI: <https://doi.org/10.1115/1.3601206>
- [4] Stefan-Dan Pastrama, et al, "Weight functions from finite element displacements," *International Journal of Pressure Vessels and Piping*, vol. 75, pp. 229-236, 1998. DOI: [https://doi.org/10.1016/S0308-0161\(98\)00029-5](https://doi.org/10.1016/S0308-0161(98)00029-5)
- [5] A.P. Mouritz., "Introduction to Aerospace Materials," <https://www.sciencedirect.com/book/9781855739468>
- [6] A. Likeb, et al., "The determination of the stress intensity factor solutions for the new pipe-ring specimen using FEA," *Archive of Applied Mechanics*, vol. 89, No. 5, pp. 897-909, 2019. DOI: <https://doi.org/10.1007/s00419-018-1481-8>
- [7] CP. Andrasic, et al., "Dimensionless stress intensity factors for cracked thick cylinders under polynomial face loadings," *Engineering Fracture Mechanics*, vol. 19, pp. 187-193, 1984. DOI: [https://doi.org/10.1016/0013-7944\(84\)90078-X](https://doi.org/10.1016/0013-7944(84)90078-X)
- [8] H. Tada, et al., "The analysis of cracks handbook," *New York: ASME Press*, vol. 2. No.1, 2000
- [9] F. Delale, et al., "Stress intensity factors in a hollow cylinder containing a radial crack," *International Journal of Fracture*, vol. 20, pp. 251-265, 1982.
- [10] A.A. Wells., "Unstable crack propagation in metals: cleavage and fast fracture," *Proceedings of the crack propagation symposium*. 1. Paper 84, 1961. Cranfield, UK
- [11] N.K. Mukhopadhyay., " Further considerations in modified crack closure integral based computation of stress intensity factor in BEM," *Engineering Fracture Mechanics*, vol. 59, No. 3, pp. 269-279, 1998. DOI: [https://doi.org/10.1016/S0013-7944\(97\)00135-5](https://doi.org/10.1016/S0013-7944(97)00135-5)
- [12] Oudrane R, Hamouda M, Aour B. The Thermal Transfers of a Habitable Envelope in an Extremely Dry Area and These Effects on Thermal Comfort. *Algerian Journal of Renewable Energy and Sustainable Development*, 2019, 1(1),79-91. <https://doi.org/10.46657/ajresd.2019.1.1.8>
- [13] A. Djebli, et al., "A 3D analysis of crack-front shape of asymmetric repaired aluminum panels with composite patches." *Frattura ed Integrità Strutturale*, vol. 13, pp. 547-556, 2019. DOI:<https://doi.org/10.3221/IGF-ESIS.49.51>.
- [14] R.S. Barsoum., "On the Use of Isoparametric Elements in Linear Fracture Mechanics," *International Journal for Numerical Methods in Engineering*, vol. 10, pp. 25-37, 1976. DOI:<https://doi.org/10.1002/nme.1620100103>
- [15] N.D. Hung, et al., "The computation of 2-d stress intensity factors using hybrid mongrel displacement finite elements," *Engineering Fracture Mechanics*, vol. 38, pp. 197-205, 1991. DOI: [https://doi.org/10.1016/0013-7944\(91\)90082-C](https://doi.org/10.1016/0013-7944(91)90082-C).
- [16] A. Baltach, et al., "Numerical Analysis of Asymmetrically Bonded Composite Patch Repair and Effect of In-Plane Skewed Crack Front on the SIF," *International Journal of Engineering Research in Africa*, vol. 30, pp. 11-22. Trans Tech Publications Ltd, 2017. DOI: <https://doi.org/10.4028/www.scientific.net/JERA.30.11>
- [17] M. Treifi, et al., "Strain energy approach to compute stress intensity factors for isotropic homogeneous and bi-material V-notches," *International Journal of Solids and Structures*, vol. 50, pp. 2196-2212, 2013. DOI: <https://doi.org/10.1016/j.ijsolstr.2013.03.011>
- [18] Z.J. Yang, et al., "Efficient evaluation of stress intensity factors using virtual crack extension technique," *Computers & Structures*, vol. 79, pp. 2705-2715, 2001. DOI: [https://doi.org/10.1016/S0045-7949\(01\)00146-8](https://doi.org/10.1016/S0045-7949(01)00146-8)

#### How to cite this paper:

Bendouba M, Djebli A, Baltach A, Benhamena A, Boukhelif A, Aid A. The Effect of the Size and Position of the Crack on the Normalized Stress Intensity Factor. *Algerian Journal of Renewable Energy and Sustainable Development*, 2020, 2(1),1-8. <https://doi.org/10.46657/ajresd.2020.2.1.1>