A family of adaptive filtering algorithms based on the variable step size

Islam Hassani[1]*, Rédha Bendoumia[2]

[1]Signal Processing and Image Laboratory (LATSI), Department of electronics, University of Blida 1, Route of Soumaa
B.P. 270, Blida 09000, Algeria
[2]Detection Information and Communication Laboratory (DIC), Department of electronics, University of Blida 1, Route of Soumaa
B.P. 270, Blida 09000, Algeria
*Corresponding author; Email: islam.hassani@univ-blida.dz

Article Info

Article history:

Received 27 Septmber, 2021
Revised 28 November 2021
Accepted 14 December , 2021

Keywords:

Adaptive filtering
NLMS
Variable Step Size
Acoustic Echo Cancellation

ABSTRACT

In this paper, we present the most used adaptive filtering algorithms such as Least Mean Square (LMS) and its normalized version NLMS with their advantages and drawbacks, and then show how the Variable Step Size (VSS) algorithms have been proposed to slove problems coming from the fixed step size. Series of simulations have been carried out under different effects such as: the size of the adaptive filter and different values of step size to validate the good behaviour of the four presented VSS based algorithms over the classical adaptive filtering algorithms with fixed step size. Also, results have confirmed the superiority of VSS based algorithms in terms of convergence speed with almost identical computational complexity.

I. Introduction

Hands-free communication systems can be considerably altered by ambient noise and by the phenomenon of acoustic echo. Noise reduction and echo cancellation processing are essential to ensure good quality of communication [1–4]. The quality of a telephone signal degrades enormously with distance and is greatly disturbed by the presence of noise and acoustic echo. The echo phenomenon, which is the reverberation of the signal during transmission, generally poses a problem in all “PC to Telephone” or “Telephone to Telephone” type communications.

Acoustic echo cancellation is a problem of identifying a linear system excited by a known reference signal (speech signal); The problem will be more complicated by the fact that the excitation signal and the echo path are non-stationary [5]. To resolve these problems, we use an echo canceller where the identification of the impulse response of the echo path is done by a digital filter with Finite Impulse Response (FIR) using for example, the stochastic gradient algorithms such as Least Mean Squares (LMS) [6], its normalized version NLMS (Normalized LMS) [7] and the Affine Projection algorithms (AP) [8]. Despite all the solutions found for the echo cancellation problem, some difficulties remain to be overcome. We can cite:

- The impulse response modeling the echo path is very long (number of samples).
- Input signals (usually speech signals), are not stationary.
- In addition to these difficulties, there is the case of double talk (two people emit simultaneously a signal).

The most used adaptive filtering algorithms are the LMS and the NLMS, because of for their simplicity of implementation and their robustness. Among the characteristics of the LMS algorithm is that its convergence rate depends on the length of the filter to be adapted and also on the correlation of the input signal of the filter to be adapted. Recently, several algorithms with a variable step size have been proposed to solve difficulties shown in LMS family, using several types of criteria such as the Variable Step-Size LMS (VSS-NLMS) algorithm [9] is updated by the instantaneous squared error, the Robust Variable Step size (RVS-LMS) algorithm is updated by the autocorrelation of the output error signal[10]. Other algorithms have been proposed such as the NLMS and APA with a variable step (VSS-NLMS and VSS-APA) [9]. All of these algorithms can theoretically be used for the cancellation of fluctuations that are found in the error signal when using a very large step-size [11].

II. Principle of the adaptive filtering

The principle of adaptive filtering or the identification of an unknown system is presented in figure (1). The unknown system $\mathbf{h} = [h(0)\ h(1) \ldots h(M-1)]$ is modeled exactly by a FIR filter of M coefficients. The adaptive filter is represented by its vector $\mathbf{w}(n)$ is ideally of a size equal to the size of the unknown system and has the role of providing an estimate $y(n)$ of the unknown signal $d(n)$, as a result the estimate is obtained by a convolution of the signal $x(n)$ and the adaptive filter $\mathbf{w}(n)$ [12, 13].

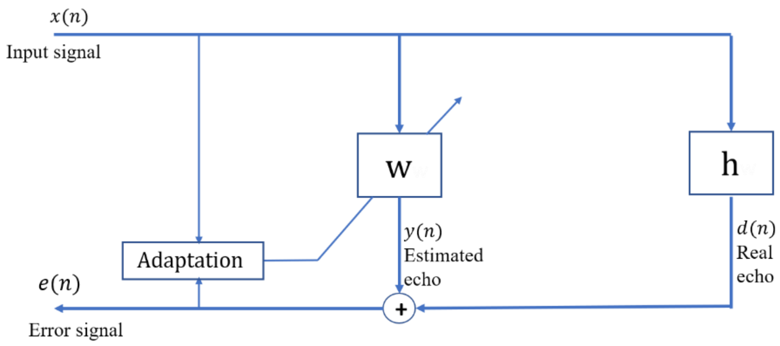

Figure 1: Principle of system identification by adaptive filtering.

The adaptive filtering technique is typically achieved by two steps:

1. A filtering step which makes it possible to obtain an estimate of the unknown system by convolving the input signal with the coefficients of the adaptive filter. The estimation error is then used in the adaptation part to update the filter coefficients.

2. An adaptation step that adjusts the coefficients of the adaptive filter according to a given algorithm.

The adaptive filtering algorithm makes it possible to calculate the coefficients of the filter $\mathbf{w}(n)$ so that the difference between the signal $d(n)$ and the current output of the filter $y(n)$ is minimized within a previously predefined statistical criterion. In general, the adaptation algorithm is presented in the following vector form:

$$\begin{bmatrix} Vector\ of \\ New \\ coefficients \\ of\ the\ filter \end{bmatrix} = \begin{bmatrix} Vector\ of \\ Old \\ coefficients \\ of\ the\ filter \end{bmatrix} + (step\ size)\begin{pmatrix} Sample\ of \\ error\ signal \end{pmatrix}\begin{bmatrix} Vector \\ of\ input \\ signal \end{bmatrix} \tag{1}$$

III. Adaptive filtering algorithms

In this section, the most used adaptive filtering algorithms in the field of acoustic echo cancellation are presented, such as the stochastic gradient and affine projection family. We are interested exactly in the LMS, NLMS and APA algorithms.

III.1 Stochastic gradient algorithm

The stochastic gradient algorithm, or Least Mean Square (LMS) is an approximation of the deterministic gradient algorithm. This algorithm has been proposed by Bernard Widrow and Marcian Hoff in 1960 [6].
The idea of the algorithms of type LMS is to replace the statistical mean of the deterministic gradient algorithm by its instantaneous value.
Autocorrelation matrix : $\mathbf{R} = E\{x(n)x(n)\}$
Cross-correlation vector: $\mathbf{r} = E\{x(n)d(n)\}$
Since $\mathbf{R}$ and $\mathbf{r}$ are unknown, these quantities will be replaced by its estimates $\widetilde{\mathbf{R}}$ and $\tilde{\mathbf{r}}$ at each instant n, as follows:

$$\widetilde{\mathbf{R}} = \mathbf{x}(n)\mathbf{x}^T(n) \quad (2)$$

$$\tilde{\mathbf{r}} = \mathbf{x}(n)d(n) \quad (3)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{LMS}[\tilde{\mathbf{r}}(n) - \widetilde{\mathbf{R}}(n)\mathbf{w}(n)] \quad (4)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{LMS}\,\mathbf{x}(n)[d(n) - \mathbf{x}^T(n)\mathbf{w}(n)] \quad (5)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{LMS}\,\mathbf{x}(n)e(n) \quad (6)$$

We notice that $\mathbf{w}(n)$ is a random variable depending at each iteration n on random processes $x(n)$ and $d(n)$.
Where: $e(n) = d(n) - y(n)$ and $y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
μ_{LMS} is a fixed step size of the algorithm.
The LMS algorithm is the most widely used algorithm in adaptive filtering applications, due to its simplicity and its reduced complexity. This algorithm requires only $2M + 1$ multiplications and $2M$ additions per iteration for a filter length M (See Table 1) [14].

Table 1: The required operations for the LMS algorithm.

Adaptation of LMS algorithm	Number of additions	Number of multiplications
$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n)$	M	M
$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{LMS}\,\mathbf{x}(n)e(n)$	M	$L+1$
Total per iteration	$2M$	$2M+1$

III.2 The Normalized LMS (NLMS) algorithm

One of the family of the LMS algorithm, called Normalized Least Mean Square (NLMS) algorithm [7], avoids the drawback of the LMS algorithm, where the adaptation gain is normalized by the power of the input signal $x(n)$. The major problem of the LMS algorithm is in the case of the power of the input signal changes over time, as a result, the step size between two adjacent coefficients changes also, this will affect the convergence speed. Therefore, the normalization is a solution to solve this problem. Then, to avoid division by zero in case of small values of the power of the input signal, we introduce a small parameter δ, where $\delta > 0$ is a regularization parameter. The update of the adaptive filter coefficients is done by the following equation [15, 16]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu_{NLMS}}{\mathbf{x}^T(n)\mathbf{x}(n)+\delta} e(n)\mathbf{x}(n) \quad (7)$$

The convergence of this algorithm is guaranteed for a step size $0 < \mu_{NLMS} \leq 2$.

The advantage of the NLMS algorithm compared to the LMS algorithm is to make the algorithm independent of the variance of the input signal. However, the distribution of the eigenvalues of the autocorrelation matrix of the input signal is not modified. This implies precisely the same dependence, in both cases, of the convergence on the statistics of the input signal. For stationary signals such as white noise or non-stationary signals such as speech, the NLMS algorithm brings a significant improvement on the convergence rate compared to the LMS thanks to the normalization of the step size. This algorithm may be more complex than the LMS algorithm but it is still one of the simplest algorithms to implement. It is often used in echo cancellation applications with its different versions presented in the following sections.

One of the disadvantages of the NLMS algorithm compared to the LMS is the increase of computational complexity in terms of the number of multiplications, as shown in Table 2.

Table 2: The required operations for the NLMS algorithm.

Adaptation of NLMS algorithm	Number of additions	Number of multiplications
$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n)$	M	M
$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu_{NLMS}}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta} e(n)\mathbf{x}(n)$	M+1	$2M + 1$
Total per iteration	2M+1	$3M + 1$

III.3 The Variable Step Size (VSS) algorithms

In the classical LMS algorithm presented by equation (6), the length of the filter is important. The condition of stability and convergence in this case, can be expressed as:

$$0 < \mu_{LMS} < \frac{1}{\lambda_{max}} \tag{8}$$

where λ_{max} represent the maximum value of the power spectral density of input signals.

According to several studies, the choice of the step size is essential for a proper functioning of the algorithms presented in the previous section, because:

- The larger the step size, the faster convergence speed, wich results to a higher variance with significant fluctuations around the mean trajectory.
- The lower the step size, the slower convergence speed, but with a lower variance with a deterministic trajectory.

To solve this tradeoff of the convergence speed and the large fluctuations, several algorithms with variable step size have been proposed, such as VSS-LMS, VSS-NLMS and VSS-APA [17–20].

The principle of the VSS based algorithms is to choose a large step size at the begening of adaptation in order to get a high convergence speed, and after that, more the error decreases more the step size is reduced in order to obtain a better precision. For the VSS-LMS algorithm, the filter update equation is in the following form:

$$\begin{bmatrix} Vector\ of \\ New \\ coefficients \\ of\ the\ filter \end{bmatrix} = \begin{bmatrix} Vector\ of \\ Old \\ coefficients \\ of\ the\ filter \end{bmatrix} + (variable\ step\ size) \begin{pmatrix} Sample\ of \\ error\ signal \end{pmatrix} \begin{bmatrix} Vector \\ of\ input \\ signal \end{bmatrix} \tag{9}$$

In the rest of this section, we will introduce some adaptive filtering algorithms with the variable step size.

Algorithm 1

This algorithm uses instantaneous squared errors to update the value of μ. When the error is large, μ will increase, which leads to a fast convergence rate; when the error decreases, μ becomes smaller, the residual variance becomes small.

The adaptation step update μ is given by the following equation:

$$\mu(n+1) = \alpha\, \mu(n) + \gamma\, e^2(n) \tag{10}$$

where α and γ are control parameters, given by: $0 < \alpha < 1$ and $\gamma > 0$

to ensure a proper functioning of the VSS-LMS algorithm for each iteration μ must be bounded between $[\mu_{min}, \mu_{max}]$.

Hence the update equation of the adaptive filter is given by:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu(n)\mathbf{x}(n)e(n) \tag{11}$$

Algorithm 2

This algorithm is derived from the previous algorithm, where the step size is adjusted using the autocorrelation between $e(n)$ and its past value $e(n-1)$. The variable step size is evaluated by the following equation:

$$\mu(n+1) = \alpha\,\mu(n) + \gamma\,p^2(n) \tag{12}$$

where

$$p(n) = \beta\,p(n-1) + (1-\beta)\,e(n)\,e(n-1) \tag{13}$$

and β is an exponential forgetting factor $0 < \beta < 1$.

Algorithm 3

In this algorithm, the step size is evaluated by the following equation:

$$\mu(n+1) = \alpha\,\mu(n) + \gamma\,p(n) \tag{14}$$

As in algorithm 2, the step size is adjusted using the autocorrelation between $e(n)$ and its past value $e(n-1)$. The proposed algorithm 3 has the same idea but using the autocorrelation function between the M past values of the error signal (with M being the length of the impulse response). Considering $p(n)$ as the estimate of the mean square correlation function between $e(n)$ and its past values $e(n-1), e(n-2), \ldots, e(n-M)$. So, the update equation for $p(n)$ is given by:

$$p(n) = \beta\,p(n-1) + (1-\beta)\sum_{i=1}^{M}[e(n)\;\;e(n-i)]^2 \tag{15}$$

Algorithm 4

In the algorithm APA with variable step size, a vector of error is used to adjust the variable step size $\mu(n)$. The update equation of the adaptive filter coefficients for the APA is given by:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\,\mathbf{X}(n)[\delta\mathbf{I} + \mathbf{X}^T(n)\mathbf{X}(n)]^{-1}\mathbf{e}(n) \tag{16}$$

The variable step size is $\mu(n)$ is given by the following equation:

$$\mu(n) = \mu_{max}\frac{\|\mathbf{p}(n)\|^2}{\|\mathbf{p}(n)\|^2+\delta_2} \tag{17}$$

where δ_2 is a positive number proportional to the projection order k, $\mu_{max} < 2$ and $\mathbf{p}(n)$ is a vector of dimension of (M x 1) which is given by:

$$\mathbf{p}(n) = \beta\,\mathbf{p}(n-1) + (1-\beta)\,\mathbf{X}(n)\,(\mathbf{X}^T(n)\,\mathbf{X}(n) + \delta_1\mathrm{I})^{-1}\mathbf{e}(n) \tag{18}$$

The NLMS algorithm is a special case of the VSS-APA with projection order of $k = 1$.

IV. Simulation results

In this part, we will give simulation results of some adaptive filtering algorithms LMS and NLMS. Also, we will test and compare the performances of these algorithms and the algorithms of a variable step size. Several simulations will be carried out which take into account different effects such as: the filter size and the step size (fixed and variable). As a measure of performance evaluation, we use the Mean Square Error (MSE) defined as:

$$MSE(dB) = 10\log_{10}(< e^2(n) >) \tag{19}$$

where $<.>$ denotes a time average over 256 samples.

IV.1 Signals and systems

The signals used in the simulations that we will detail in this paper are:

A white Gaussian noise which is a realization of a random process in which the power spectral density is the same for all frequencies, and is mainly used to check the numerical stability of the algorithms.

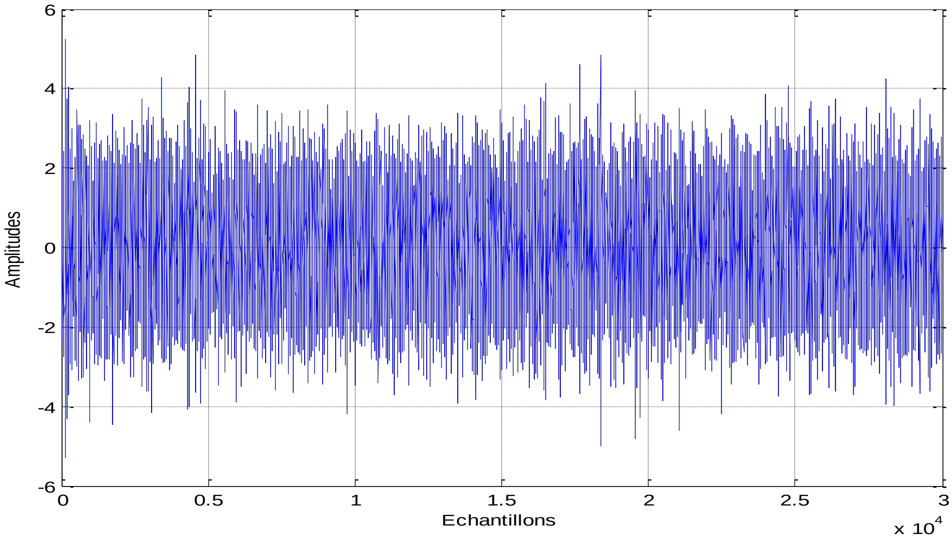

Figure 2: White Gaussian noise signal.

Two impulse responses are used to model the echo path in these simulations, with lengths of 128 and 256 samples.

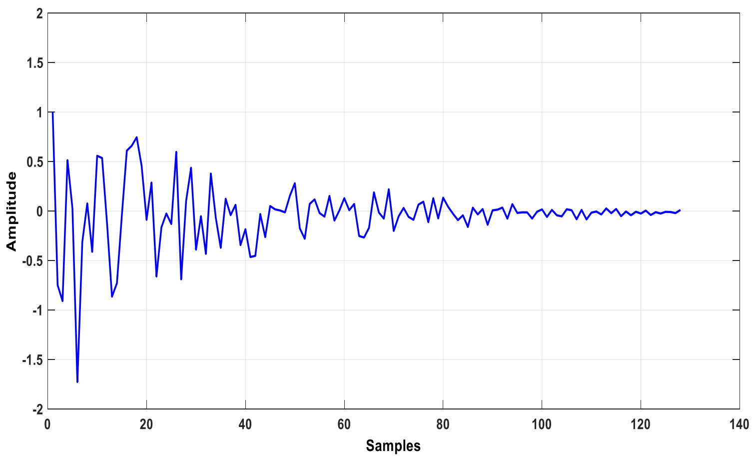

Figure 3: The used impulse response of length 128.

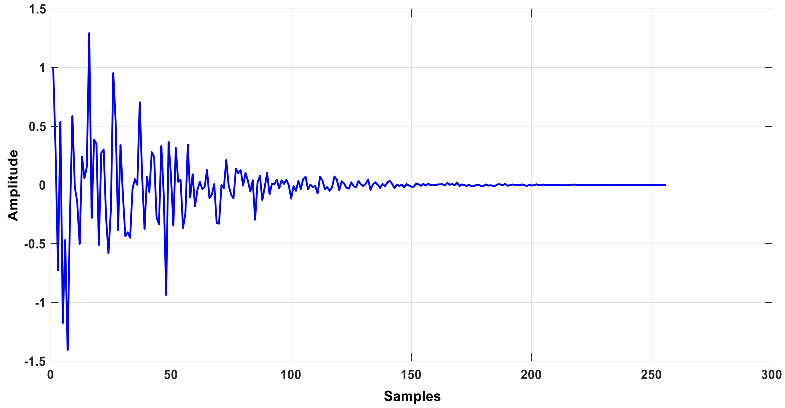

Figure 4: The used impulse response of length 256.

IV.2 The LMS and NLMS algorithms

In this subsection, we made a series of tests of the stochastic gradient algorithms (LMS and NLMS) in the context of system identification. These two algorithms give effective results to solve the problem of AEC and system identification. The main advantage of these algorithms is their simplicity of implementation. However, the main drawback is the adjustment of the step size μ to ensure better performances. For this purpose, we have fixed the size of the filter once $L = 128$ and the other time at 256 and for each size we varied the step size μ. We obtained the results presented in the following figures:

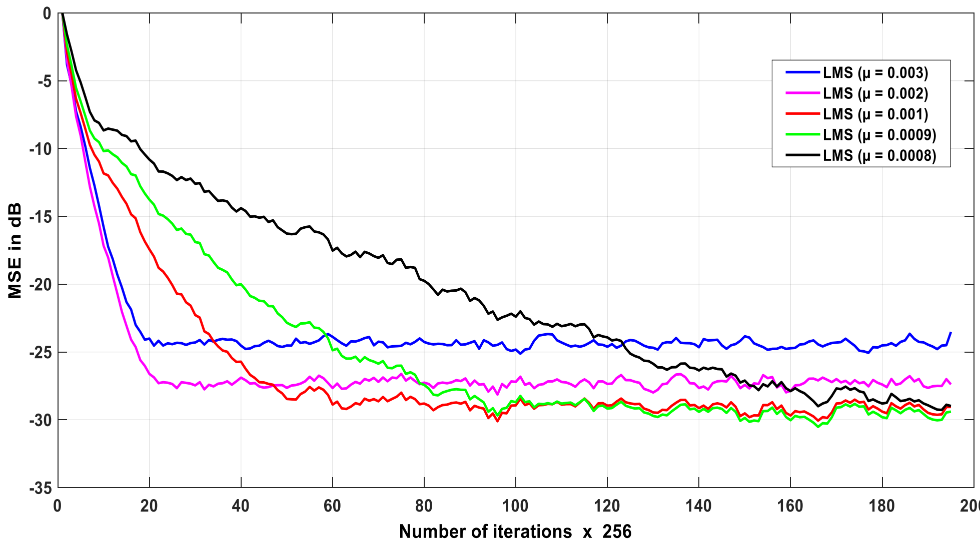

Figure 5: MSE of the LMS algorithm for different step size values μ. Filter length $M = 256$, SNR= 30 dB, input signal: White Gaussian noise.

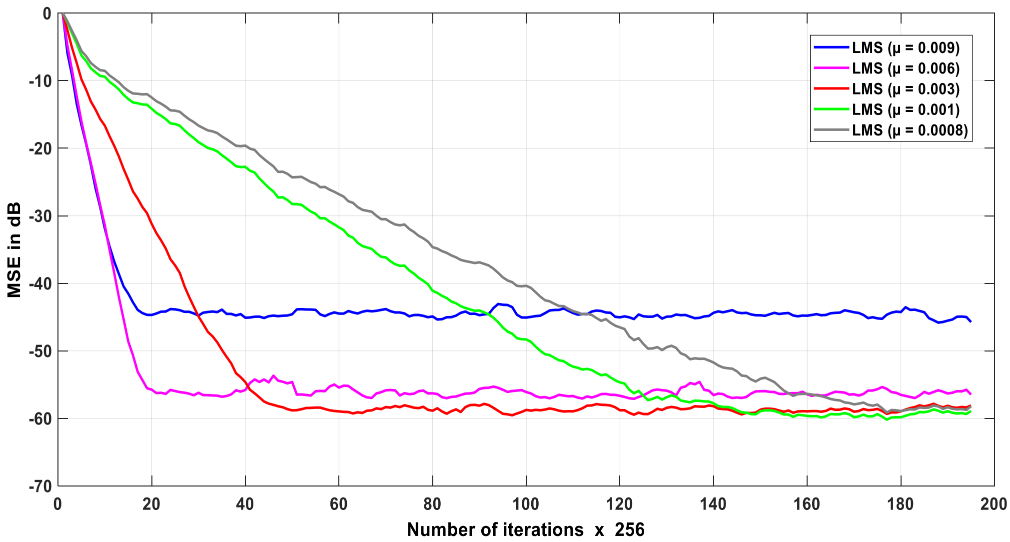

Figure 6: MSE of the LMS algorithm for different step size values μ. Filter length $M = 128$, SNR= 60 dB, input signal: White Gaussian noise.

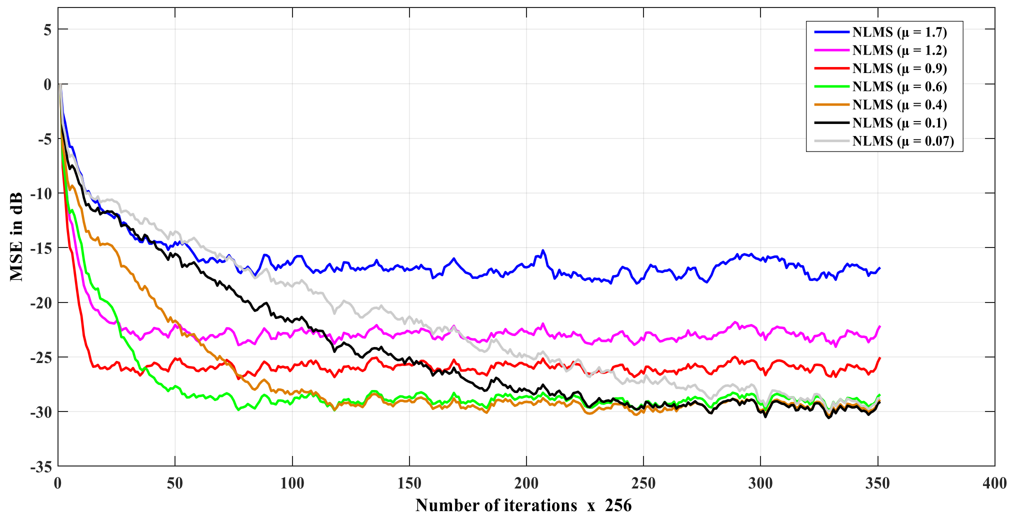

Figure 7: MSE of the NLMS algorithm for different step size values μ. Filter length $M = 256$, SNR= 30 dB, input signal: White Gaussian noise.

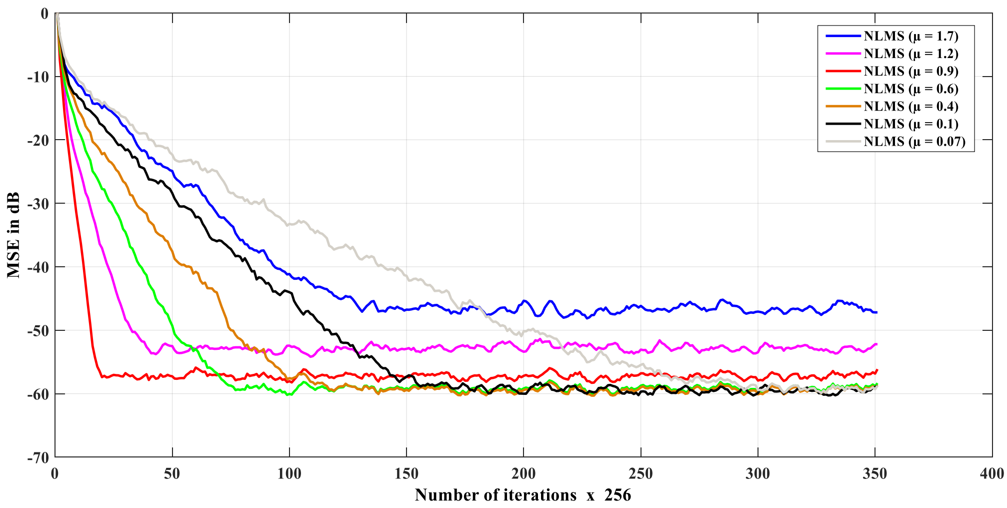

Figure 8: MSE of the NLMS algorithm for different step size values μ. Filter length $M = 128$, SNR= 60 dB, input signal: White Gaussian noise.

The above figures represent the temporal evolution of the MSE of the two algorithms LMS and NLMS, for different values of parameter μ (step size). These results are obtained from 50,000 and 90,000 iterations and using a filter of size of 128 and 256. Through the obtained results we have clearly observed that the choice of this parameter is very important to obtain an adaptation more or less acceptable. If the step size μ is very large, the convergence speed is very fast, but there are significant fluctuations around the mean trajectory. When using a very small step size μ, the convergence speed is very slow, but the fluctuations are small. As a conclusion, the increase of the step size affects the stability as well as the convergence speed, then the coefficients of the adaptive filter converge towards the desired values with a significant step, which does not always allow these coefficients to take the desired values with more precision, there will therefore be more oscillation of the error $e(n)$ around zero. To solve this problem, researchers have proposed to use a variable step size. In the next subsection, we will see the simulation results of adaptive filtering algorithms with a variable step size.

IV.3 The variable step size (VSS) adaptive filtering algorithms

In the following table, we will present four adaptive filtering algorithms with variable step size which are studied previously.

Table 3: Four adaptive filtering algorithms with variable step size.

Algorithms	***Variable step size***
Algorithm 1	$\mu(n+1) = \alpha\ \mu(n) + \gamma\ p^2(n)$ $p(n) = e(n)$
Algorithm 2	$\mu(n+1) = \alpha\ \mu(n) + \gamma\ p^2(n)$ $p(n) = \beta\ p(n-1) + (1-\beta)\, e(n)\ e(n-1)$
Algorithme3	$\mu(n+1) = \alpha\ \mu(n) + \gamma\ p(n)$

	$p(n) = \beta\ p(n-1) + (1-\beta) \sum_{i=1}^{M} [e(n)\ e(n-i)]^2$
Algorithm 4	$\mu(n) = \mu_{max} \cdot \frac{\lVert \mathbf{p}(n) \rVert^2}{\lVert \mathbf{p}(n) \rVert^2 + \delta_2}$ $\mathbf{p}(n) = \beta\ \mathbf{p}(n-1) + (1-\beta)\ \mathbf{x}(n)\ \left(\mathbf{x}^T(n)\ \mathbf{x}(n) + \delta_1 I\right)^{-1} e(n)$

In the following, we will present comparative results between the adaptive filtering algorithms with a fixed step size NLMS and variable step size VSS-NLMS which have been studied and discussed before.

Algorithm 1

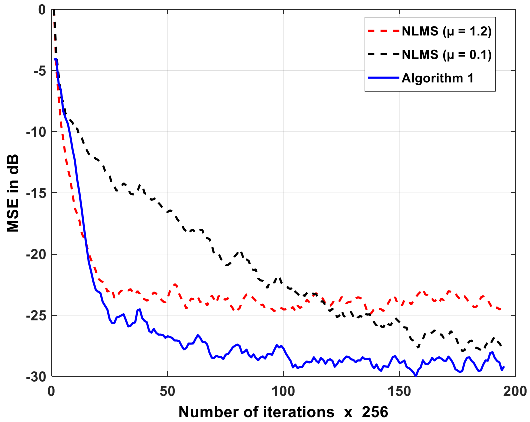

Figure 9: MSE for NLMS and Algorithm 1

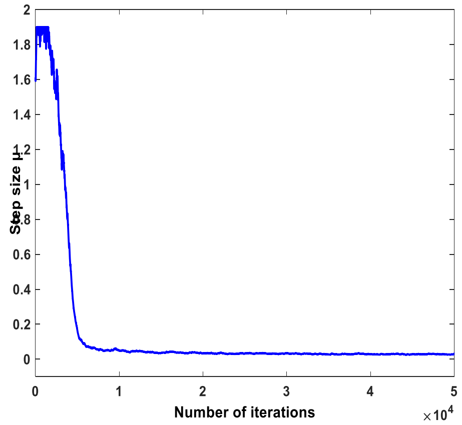

Figure 10: Evaluation of the step size $\mu(n)$

With length $M = 256$ and SNR= 30 dB. Input signal: White Gaussian noise. Algorithm 1 ($\alpha = 0.9991$; $\gamma = 0.005$; $\mu_0 = 1.6$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00000000009$)

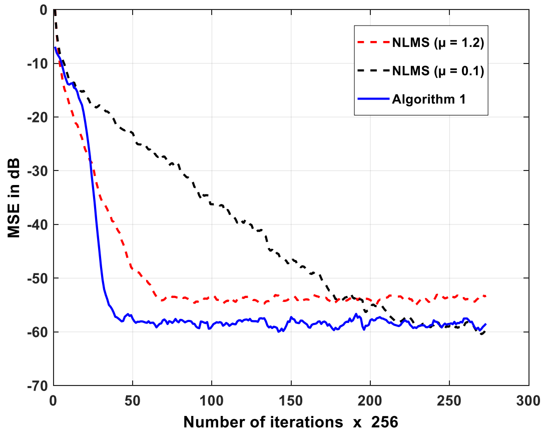

Figure 11: MSE for NLMS and Algorithm 1

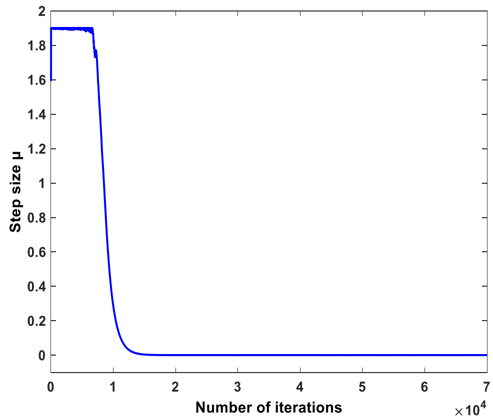

Figure 12: Evaluation of the step size $\mu(n)$

With length $M = 128$ and SNR= 60 dB. Input signal: White Gaussian noise. Algorithm 1 ($\alpha = 0.997$; $\gamma = 0.002$; $\mu_0 = 1.6$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00000000009$)

Algorithm 2

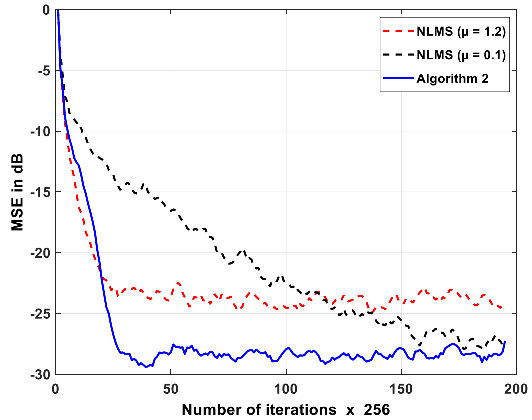

Figure 13: MSE for NLMS and Algorithm 2

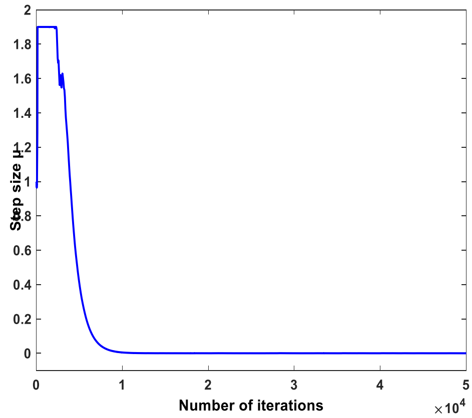

Figure 14: Evaluation of the step size $\mu(n)$

With length $M = 256$ and SNR= 30 dB. Input signal: White Gaussian noise. Algorithm 2 ($\alpha = 0.9991$; $\gamma = 0.02$; $\beta = 0.99$; $\mu_0 = 1$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00009$)

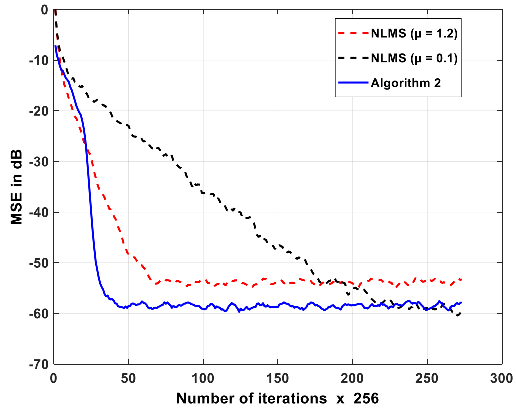

Figure 15: MSE for NLMS and Algorithm 2

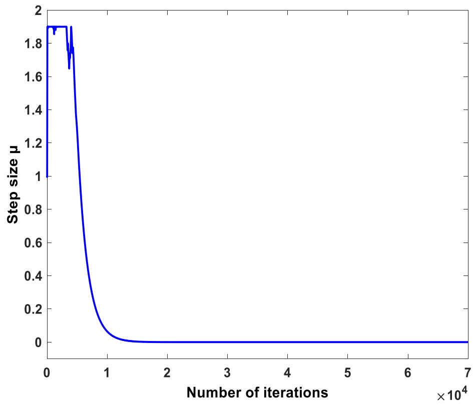

Figure 16: Evaluation of the step size $\mu(n)$

With length $M = 128$ and SNR= 60 dB. Input signal: White Gaussian noise. Algorithm 2 ($\alpha = 0.9994$; $\gamma = 0.9$; $\beta = 0.99$ $\mu_0 = 1$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00009$)

Algorithm 3

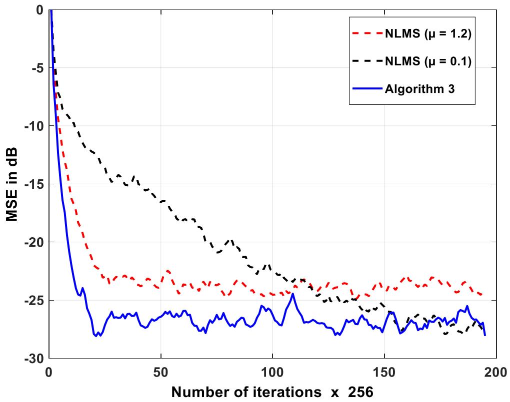

Figure 17: MSE for NLMS and Algorithm 2

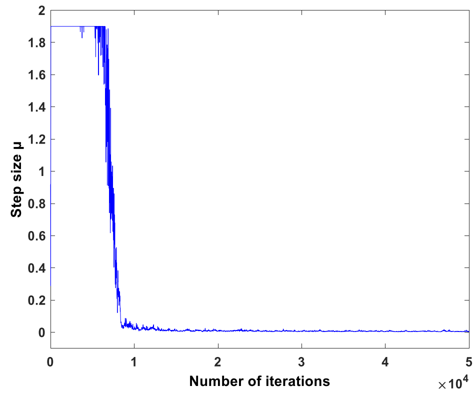

Figure 18: Evaluation of the step size $\mu(n)$

With length $M = 256$ and SNR= 30 dB. Input signal: White Gaussian noise. Algorithm 3 ($\alpha = 0.92$; $\gamma = 0.4\ 10^{-3}$; $\beta = 0.9$; $\mu_0 = 1$; $\mu_{max} = 1.9$; $\mu_{min} = 0.0009$)

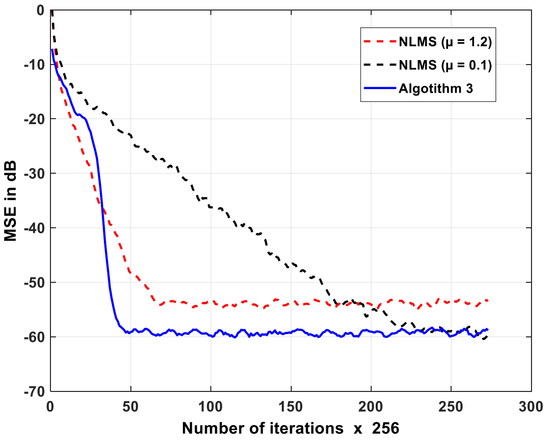

Figure 19: MSE for NLMS and Algorithm 3

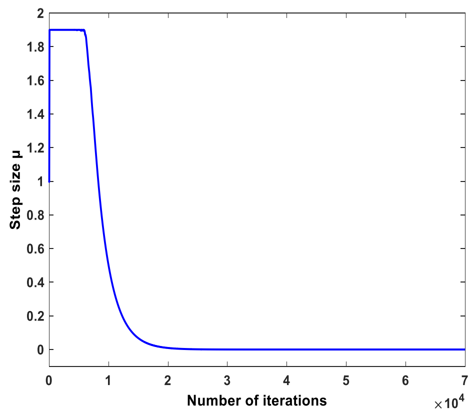

Figure 20: Evaluation of the step size $\mu(n)$

With length $M = 128$ and SNR= 60 dB. Input signal: White Gaussian noise. Algorithm 3 ($\alpha = 0.9996$; $\gamma = 0.8\ 10^{-4}$; $\beta = 0.9$ $\mu_0 = 1$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00000000009$)

Algorithm 4

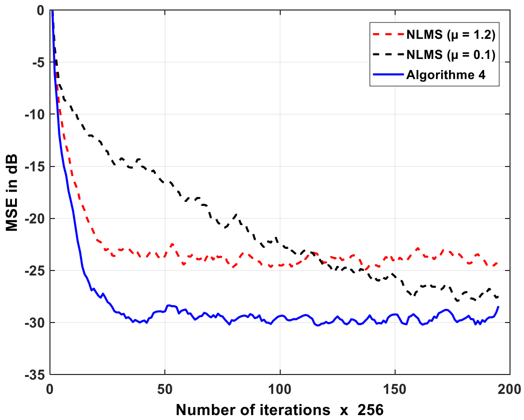

Figure 21: MSE for NLMS and Algorithm 4

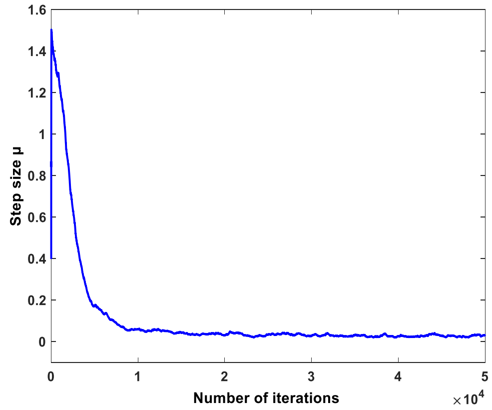

Figure 22: Evaluation of the step size $\mu(n)$

With length $M = 256$ and SNR= 30 dB. Input signal: White Gaussian noise. Algorithm 4 ($\alpha = 0.9995$; $\gamma = 0.8$; $\beta = 0.99$; $\mu_0 = 0.9$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00009$; $c = 10^{-6}$)

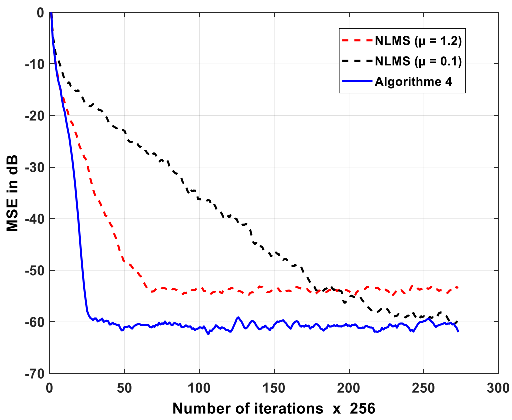

Figure 23: MSE for NLMS and Algorithm 4

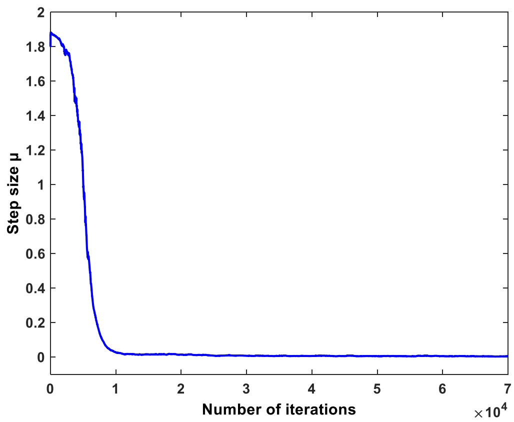

Figure 24: Evaluation of the step size $\mu(n)$

With length $M = 128$ and SNR= 60 dB. Input signal: White Gaussian noise. Algorithm 4 ($\alpha = 0.9995$; $\gamma = 0.8$; $\beta = 0.99$; $\mu_0 = 0.9$; $\mu_{max} = 1.9$; $\mu_{min} = 0.00009$; $c = 10^{-6}$)

IV.4 Comparison of NLMS algorithm with four VSS algorithms

In this subsection, parameters of the VSS algorithms are summarized in the following table.

Table 4: The parameters of the VSS based algorithms

Algorithms	Parameters
Algorithm 1	$\alpha = 0.997$; $\gamma = 0.002$;
Algorithm 2	$\alpha = 0.9991$; $\gamma = 0.02$; $\beta = 0.99$;
Algorithm 3	$\alpha = 0.9991$; $\gamma = 5\ 10^{-7}$; $\beta = 0.9$;
Algorithm 4	$\alpha = 0.9995$; $\gamma = 0.8$; $\beta = 0.99$; $c = 5\ 10^{-7}$;

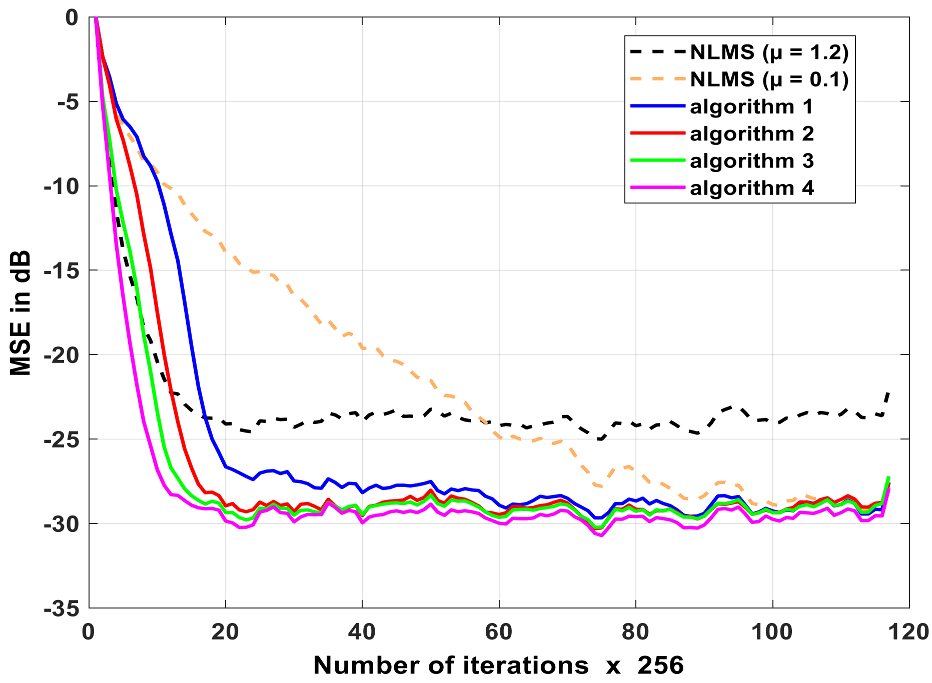

Figure 25: MSE of the NLMS algorithm and four VSS algorithms. Filter length $M = 128$, SNR=30 dB, input signal: White Gaussian noise.

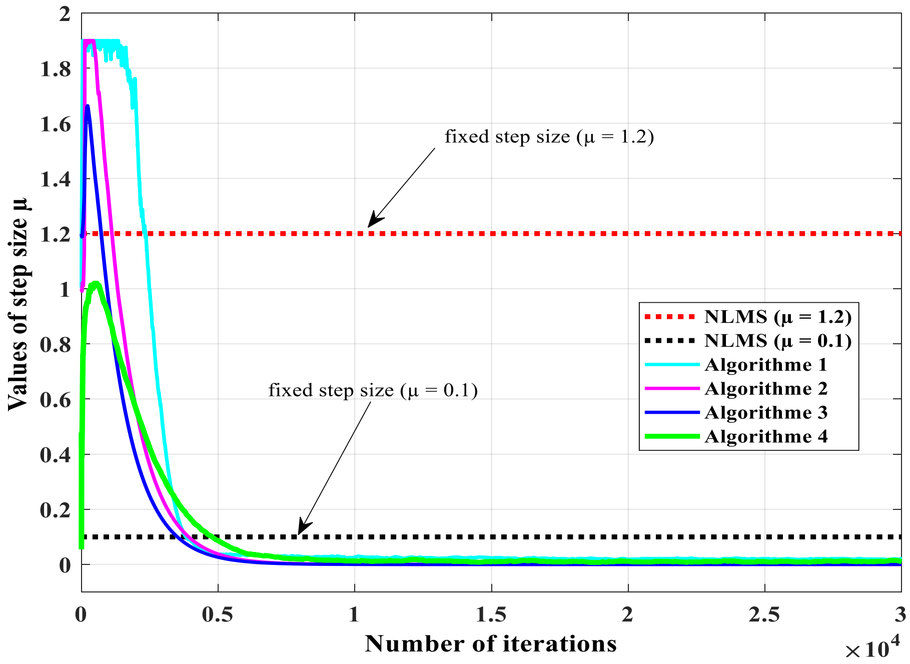

Figure 26: Evaluation of the step size $\mu(n)$

From the results of comparative simulations illustrated in the above figures, we well notice that the NLMS algorithm converges better when the adaptation step μ is chosen large, but with significant fluctuations around the mean trajectory. The four algorithms with variable step size give good results for AEC and system identification. We can say that a VSS aims to improve the convergence speed as shown in the previous figures, with a lower variance. So, when the error is large, μ increase, which leads to a fast convergence speed; when the error decreases, μ becomes smaller, the fluctuations become small also.

V. Conclusion

In this paper, we have presented several adaptive filtering algorithms with a Variable Step Size (VSS-LMS, VSS-NLMS and VSS-APA). Some algorithms based on the energy of the input signal and others used the error signal. These algorithms were tested for system identification. The results obtained show the superiority of the VSS based algorithms compared to algorithms with a fixed step size, either for the improvement of the convergence speed (fast) or the distortion at the output (less fluctuations) by keeping almost the same computational complexity. Among the algorithms presented, the fourth algorithm (VSS-NLMS) is the best compared to the others. Recently, this type of algorithms is widely used in all applications of adaptive filtering. VSS algorithms can be considered as a solution to solve the convergence speed problem in some algorithms (LMS and NLMS) and the fluctuations present in the error signal, and to recover the speech signal without distortion in case of AEC applications.

References

1. Bendoumia, R. (2019). Efficient Sparse Blind Source Separation Algorithm for two- Channel Acoustic Noise Reduction. *Algerian Journal of Renewable Energy and Sustainable Development*, *1*(01), 31–40. https://doi.org/10.46657/AJRESD.2019.1.1.4
2. Bendoumia, R., Guessoum, A., Hassani, I., Bougheddaoui, S., & Cherif, R. (2021). New Simplified Sub-band Sparse Adaptive Algorithm for Speech Enhancement in Communication Systems. *Lecture Notes in Networks and Systems*, *361 LNNS*, 506–517. https://doi.org/10.1007/978-3-030-92038-8_51

3. Hassani, I., Benallal, A., & Bendoumia, R. (2021). Double-Talk Robust Fast Converging and Low Complexity Algorithm for Acoustic Echo Cancellation in Teleconferencing System. *Lecture Notes in Networks and Systems*, *361 LNNS*, 409–420. https://doi.org/10.1007/978-3-030-92038-8_41
4. Hassani, I., Arezki, M., & Benallal, A. (2021). The Fast-NLMS Algorithm for Acoustic Echo Cancellation with Double-Talk Detection. *Lecture Notes in Electrical Engineering*, *682*, 1271–1280. https://doi.org/10.1007/978-981-15-6403-1_90
5. Hassani, I., Arezki, M., & Benallal, A. (2020). A novel set membership fast NLMS algorithm for acoustic echo cancellation. *Applied Acoustics*, *163*. https://doi.org/10.1016/j.apacoust.2020.107210
6. B.Widrow; S.D.Stearns. (1985). *Adaptive signal processing* (1st ed.). Englewood Cliffs, NJ, Prentice Hall. Inc.
7. Arthur E Albert; Leland A. Gardner. (1967). Stochastic approximation and nonlinear regression. *MIT press*.
8. Bhotto, Md. Z. A., & Antoniou, A. (2012). Robust Set-Membership Affine-Projection Adaptive-Filtering Algorithm. *IEEE Transactions on Signal Processing*, *60*(1), 73–81. https://doi.org/10.1109/TSP.2011.2170980
9. Shin, H. C., Sayed, A. H., & Song, W. J. (2004). Variable step-size NLMS and affine projection algorithms. *IEEE Signal Processing Letters*, *11*(2 PART II), 132–135. https://doi.org/10.1109/LSP.2003.821722
10. Vega, L. R., Rey, H., Benesty, J., & Tressens, S. (2008). A new robust variable step-size NLMS algorithm. *IEEE Transactions on Signal Processing*, *56*(5), 1878–1893. https://doi.org/10.1109/TSP.2007.913142
11. Bendoumia, R. (2018). Two-channel forward NLMS algorithm combined with simple variable step-sizes for speech quality enhancement. *Analog Integrated Circuits and Signal Processing 2018 98:1*, *98*(1), 27–40. https://doi.org/10.1007/S10470-018-1269-3
12. Hassani, I., Arezki, M., & Benallal, A. (2019). Set-Membership Fast-NLMS Algorithm for Acoustic Echo Cancellation. In *2018 International Conference on Signal, Image, Vision and their Applications, SIVA 2018*. https://doi.org/10.1109/SIVA.2018.8661036
13. Hassani, I., Kedjar, A., Ramdane, M. A., Arezki, M., & Benallal, A. (2019). Fast Sparse Adaptive Filtering Algorithms for Acoustic Echo Cancellation. In *Proceedings - International Conference on Communications and Electrical Engineering, ICCEE 2018*. https://doi.org/10.1109/CCEE.2018.8634473
14. Paleologu, C., Ciochin, S., Benesty, J., & Member, S. (2008). Variable Step-Size NLMS Algorithm for Under-Modeling Acoustic Echo Cancellation. *IEEE SIGNAL PROCESSING LETTERS*, *15*, 5. https://doi.org/10.1109/LSP.2007.910276
15. Haykin, S. S. (2014). *Adaptive Filter Theory* (5th ed.). Pearson Education India.
16. Hassani, I., Kedjar, A., Ramdane, M. A., Arezki, M., & Benallal, A. (2018). Fast sparse adaptive filtering algorithms for acoustic echo cancellation. In *2018 International Conference on Communications and Electrical Engineering (ICCEE)* (pp. 1–5).
17. Huang, H. C., & Lee, J. (2012). A new variable step-size NLMS algorithm and its performance analysis. *IEEE Transactions on Signal Processing*, *60*(4), 2055–2060. https://doi.org/10.1109/TSP.2011.2181505
18. Sulyman, A. I., & Zerguine, A. (2003). Convergence and steady-state analysis of a variable step-size NLMS algorithm. *Signal Processing*, *83*(6), 1255–1273. https://doi.org/10.1016/S0165-1684(03)00044-6
19. Iqbal, M. A., & Grant, S. L. (2008). Novel variable step size NLMS algorithms for echo cancellation. *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing - Proceedings*, 241–244. https://doi.org/10.1109/ICASSP.2008.4517591
20. Rey Vega, L., Rey, H., & Benesty, J. (2010). A robust variable step-size affine projection algorithm. *Signal Processing*, *90*(9), 2806–2810. https://doi.org/10.1016/J.SIGPRO.2010.03.029