A New Iterative Method for Estimation of Carrier Frequency in Multi-Carrier Systems

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ABSTRACT

The key problem for OFDM (Orthogonal Frequency Division Multiple) systems is who to Estimate carrier frequency offset (CFO) with reduced complexity and acceptable performance. The CFO must be compensated before DFT (Discrete Fourier Transform) in order to restore data correctly and enhance the system performance. This paper, present a low complexity estimator of CFO with Semi-Blind (SB) criterion based on pilot tones and on virtual subcarriers, and with the aid of subspace based method. However, MUSIC and ESPRIT based semi-blind algorithms require a highly computational complexity. To overcome this drawback, we use Taylor's series for the first order as developed in [1]. The present methods developed in this paper are very suitable for Multi- Carrier (MC) systems when the CFO are present. Simulation results demonstrated that the semi-blind (SB) approach outperforms the blind-based approach.

I. Introduction

Orthogonal Frequency Division Multiple (OFDM) has been adopted by next Generation Broadband Wireless standards (e.g., 3GPP-LTE in Europe and WiMax in the US) due to its robustness to the channel frequency selectivity [1]. However, it has been shown that the OFDM is sensitive to carrier frequency offset (CFO) which comes from Doppler shifts or transceiver oscillator's instabilities. The CFO causes inter-carrier interference (ICI) among subcarriers, which results in severe bit error rate (BER) performance degradation. Thus, many algorithms have been proposed to estimate the CFO in OFDM for decades [2─8].

Blind CFO estimation algorithms are a class of algorithms where the CFO is estimated using the statistical properties of the received signal only, without explicit knowledge of the transmitted signal. Therefore, it does not require training sequences. In SISO-OFDM systems, blind CFO estimation algorithms usually make use of some special properties of OFDM symbols such as the cyclic prefix in the time domain and guard null subcarriers in the frequency domain.

Recently, the subspace-based blind approach has attracted much attention for CFO estimation due to its advantages of good performance and no preamble required. Such an algorithm for jointly estimating the carrier offset and the channel response has been in [8].

Liu and Tureli were the first who have used virtual subcarriers in CFO estimation [5-6]. They proposed a MUSIC-like subspace algorithm, with a search method and a search-free rooting method. The algorithm developed in [5] has excellent performance in whole band estimation range; its computational complexity is too high for practical use. When restricting CFO within a small range, Attallah simplified the cost function in [5] based on Taylor's series approximation.

The optimal subcarrier placement that minimizes the Cramer–Rao bound (CRB) of the CFO estimation was reported by Ghogho and al [9], the number of null subcarriers (NSC) and their placement are arbitrary.

In this paper, we derive a low complexity blind and semi- blind CFO estimator for OFDM systems based on Taylor's series approximation of the developed cost functions. The latter are based on virtual and pilot subcarriers.

II. System and Signal Model

In OFDM, data belonging to a single source is first divided in blocks. Then an IDFT is performed on each block and a CP is added with proper length, which must be longer than the length of the channel impulse response. In the receiver, the CP is removed and data is transformed again to the frequency domain using DFT, then a set of 1-tap equalizations is performed.

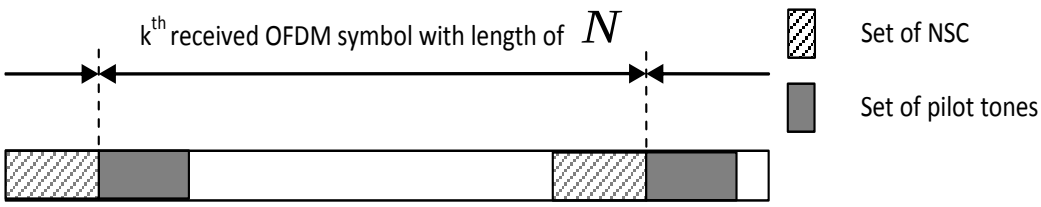

Figure 1.Base band OFDM symbol

Such multicarrier scenario completely mitigates ISI and ICI distortions as long as the channel model does not change during one OFDM symbol.

We consider an OFDM system with N subcarriers corrupted by CFO. We model the frequency selective channel as a finite impulse response FIR filters with channel impulse response $h = [h_0, \dots, h_{L-1}]^T$, where L is the channel order.

At the receiver side, assuming perfect synchronization is achieved, and after discarding the CP, the received signal can be written as:

$$y(k) = E(\varepsilon_0)F_N^H H\Phi_p x_p(k) + E(\varepsilon_0)F_N^H H\Phi_d x_d(k) + w(k) \qquad (1)$$

Where

$$E(\varepsilon_0) = diag\left(1, e^{j\varepsilon_0}, \dots, e^{j(N-1)\varepsilon_0}\right),$$

$$F_N^H = [f_0, f_1, \dots, f_{N-1}] \qquad (2)$$

F_N^H Is the $N \times N$ DFT matrix,

$$f_i = \frac{1}{\sqrt{N}}\left[1, e^{-j\frac{2\pi i}{N}}, \dots, e^{-j\frac{2\pi i(N-1)}{N}}\right] \qquad (3)$$

The channel matrix H can be defined by $H = diag\{F_L h\}$, $w(k)$ indicates the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma_w^2 I_N$. We define the index sets of pilots and data as C_p and C_d, whose size are given by N_p and N_d respectively. We define a diagonal matrix $\widetilde{\Phi}_p$ where $\left[\widetilde{\Phi}_p\right]_{i,i} = 1$ for $i \in C_p$ and $\left[\widetilde{\Phi}_p\right]_{i,i} = 0$ otherwise. Extracting the columns whose indices are included in C_p from $\widetilde{\Phi}_p$ yields the matrix $\Phi_p \in C_p^{N\times N_p}$. Similarly, $\Phi_d \in C_d^{N\times N_d}$ is defined with C_d.

x_p and x_d stand for the transmitted pilot vector of length N_p and the transmitted data vector of length N_d, respectively.

We can rewrite equation (1) as:

$$y(k) = E(\varepsilon)F_p^H H_p x_p(k) + E(\varepsilon)F_d^H H_d x_d(k) + w(k) \qquad (4)$$

Where H_p and H_d denote a diagonal matrix,

$$H_p = diag(H_0, \dots, H_{p-1}), H_d = diag(H_p, \dots, H_{N-1}).$$

For practical OFDM systems, it is usually necessary to place some null sub-carriers consecutively at both ends of the spectrum as guard bands. We call these null subcarriers the guard null subcarriers.

The optimal location of NSC and pilot tones are investigated in [10, 11], but here we put the pilot and null subcarriers consecutively at the beginning and at the end of each OFDM symbol respectively to compare the semi blind with blind methods, as illustrated in figure (1).

III. Semi-Blind CFO Estimator

The proposed method consists in the minimization of the following cost function $P(z)$

$$P(z) = \sum_{k=1}^{K}\sum_{i=1}^{L} \|F_i^H Z^{-1} y(k)\|^2$$

$$= \sum_{k=1}^{K}\sum_{i=1}^{L} \left\|F_i^H Z^{-1} E(\varepsilon_0)\{F_p^H H_p x_p(k) + F_d^H H_d x_d(k) + w(k)\}\right\|^2 \qquad (5)$$

Where K the total number of OFDM symbols used for CFO estimation, F_i^H is the ith row of the DFT matrix and $Z = diag(1, z, \dots, z^{N-1})$ with $z = e^{-j\varepsilon}$. If the number of pilot tones is zeros $N_p = 0$, the cost function reduces to that given in [5].

The algorithm proposed in [5] is shown to have a good performance as compared to Cramer-Rao Bound (CRB) and its acquisition range for CFO is much larger than that of the blind CFO estimation algorithm using the cyclic prefix [4].

A major disadvantage of this algorithm is its high computational complexity. In [1], a method is proposed to reduce the computational complexity of the method in [5-6]. This algorithm exploits the fact that the inverse diagonal matrix Z^{-1} can be rewritten as a Taylor's Series Approximation as follows:

$$Z^{-1} = e^{-j\varepsilon\frac{(N-1)}{2}} \times diag\left(e^{j\varepsilon\frac{(N-1)}{2}}, e^{j\varepsilon\frac{(N-3)}{2}}, \dots, e^{j\varepsilon\frac{(1-N)}{2}}\right)$$

$$\approx e^{-j\varepsilon\frac{(N-1)}{2}} \times \sum_{n=0}^{Q} \frac{(j\varepsilon)^n}{2^n n!} D^{-1} \qquad (6)$$

Where,

$$D^{-1} = diag\big((N-1), (N-3), (N-5), \dots, (1-N)\big)$$

Substitute (6) into (5), we get the low-complexity function

$$P_Q(\varepsilon) = \sum_{k=1}^{K}\sum_{i=1}^{L} \left\|F_i^H \left[e^{-j\varepsilon\frac{(N-1)}{2}} \times \sum_{n=0}^{Q} \frac{(j\varepsilon)^n}{2^n n!} D^{-1}\right] y(k)\right\|^2 \qquad (7)$$

We adopt the same assumptions as in [1], equation (7) can be rewrite as

$$P_Q(\varepsilon) = \sum_{l=0}^{2Q} \left(\frac{j}{2}\right)^l \varepsilon^l \sum_{m=0}^{l} \frac{(-1)^m}{(l-m)!\,(m)!} \sum_{i=1}^{L} \sum_{k=1}^{K} a_{i,l-m}(k) a_{i,m}^*(k)$$

$$= \sum_{l=0}^{2Q} c_l \varepsilon^l \qquad (8)$$

Where c_l are given by

$$c_l = \left(\frac{j}{2}\right)^l \sum_{m=0}^{l} \frac{(-1)^m}{(l-m)!(m)!} \sum_{i=1}^{L} \sum_{k=1}^{K} a_{i,l-m}(k) a_{i,m}^*(k) \qquad (9)$$

and

$$a_{i,n}(k) = F_i^H D^n y(k) \qquad (10)$$

To minimize (08), we compute its derivative with respect to ε and set it to zero i.e.

$$\frac{\partial P_{2Q}(\varepsilon)}{\partial \varepsilon} = \sum_{l=1}^{2Q} l c_l \varepsilon^{l-1} = 0 \qquad (11)$$

The estimated minimum is given by the root which, once substituted in (08), gives the smallest value for the cost function.

In practice, the CFO can be so small that only a very limited number of terms are needed for the Taylor series approximation. In this case, we can compute directly the CFO through a simple formula as follows. For$Q = 1$, the cost function becomes a polynomial equation of degree two and its derivative with respect to ε is given by

$$c_1 \varepsilon^0 + 2c_2 \varepsilon^1 = 0 \qquad (12)$$

From (12), the estimation value of CFO is given by

$$\varepsilon = -\frac{c_1}{2c_2} \qquad (13)$$

IV. Iterative Semi Blind Estimation of CFO

The performance of the semi blind CFO estimator method depends on the accuracy of the Taylor series approximation of the cost function in (8), which is determined by the number of terms used in the summation as well as the residual CFO values. As we only use $Q = 1$ (low cost) in the proposed method, the accuracy of the approximation is degraded when the actual CFO value is relatively large. As a result, there will be some performance degradation in the Mean Square Error (MSE) of the CFO estimation.

To improve the previous performance of the low complexity algorithm proposed in section 3 and based on [12], we propose a new low complexity iterative semi-blind CFO estimation algorithm:

In the first iteration, we use the proposed low complexity method derived in section 3 to obtain an initial estimate of CFOε_0. Then we compensate this CFO value in the received signal as follows.

$$\hat{y}_1(k) = C(-\varepsilon_0) y(k)$$

$$= C(-\varepsilon_0) E(\varepsilon) F_p^H H_p x_p(k) + C(-\varepsilon_0) E(\varepsilon) F_d^H H_d x_d(k) + C(-\varepsilon_0) w(k)$$

$$= E_c(\varepsilon - \varepsilon_0) F_p^H H_p x_p(k) + E_c(\varepsilon - \varepsilon_0) F_d^H H_d x_d(k) + v(k) \qquad (14)$$

Where

$$C(-\varepsilon_0) = diag\left(1, e^{-j\varepsilon_0}, \dots, e^{-j(N-1)\varepsilon_0}\right)$$

And $E_c(\varepsilon - \varepsilon_0)$ represent a new CFO matrix, which is the different between the actual and the estimated CFO value in the first iteration, $v(k)$ is statistically the same as $w(k)$ as they are both Gaussian. This means that the

noise power remains constant throughout all the iterations and no noise amplification occurs in the iterative process [12]. We repeat the same procedure iteratively for the second and third to i^{th} iteration, we will have smaller and smaller residual carrier offset to estimate and compensate as iteration goes on.

V. Simulation Results

This paragraph presents some results of the proposed semi blind CFO estimation method and compares it with the blind once. The Normalized MSE is given by:

$$NMSE = \frac{1}{N_s}\sum_{n=1}^{N_s}\left(\frac{\varepsilon - \varepsilon_0}{\omega}\right)^2$$

We consider an OFDM system with $N = 64$ subcarriers, the subcarrier spacing $\omega = 2\pi/N$, and the true frequency offset is 0.1ω. The data are randomly drawn from a BPSK. Where $N_s = 200$ represents the total number of Monte Carlo trials. Multipath fading channel considered in simulations is this used in [6]. In all the simulations, we only use 1 OFDM symbol to perform CFO estimation, i.e. K = 1.

For blind estimation of CFO, $\boldsymbol{x}_p = 0$, and the vector dimension of x_d is extended to become equal to $N_p + N_d$.

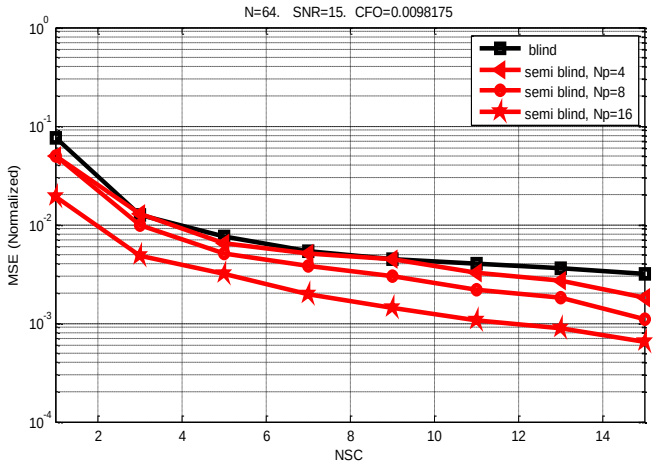

Figure. 2. MSE vs NSC

Figure.2 gives the normalized MSEs versus number of pilot tones. We remark that the semi-blind estimator outperforms the blind one as the number of pilot tones increases.

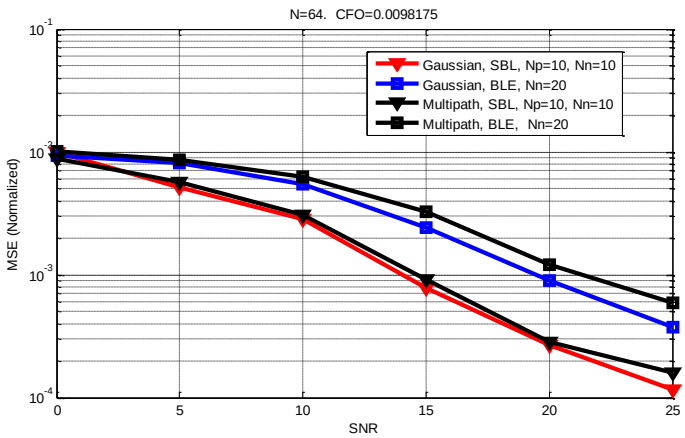

Figure 3. MSE vs SNR

Figure 3 depicts NMSE versus SNR of semi- blind and blind based CFO estimator in Gaussian and in multipath channels. In both channels, the semi-blind results are superior to those based on blind estimation.

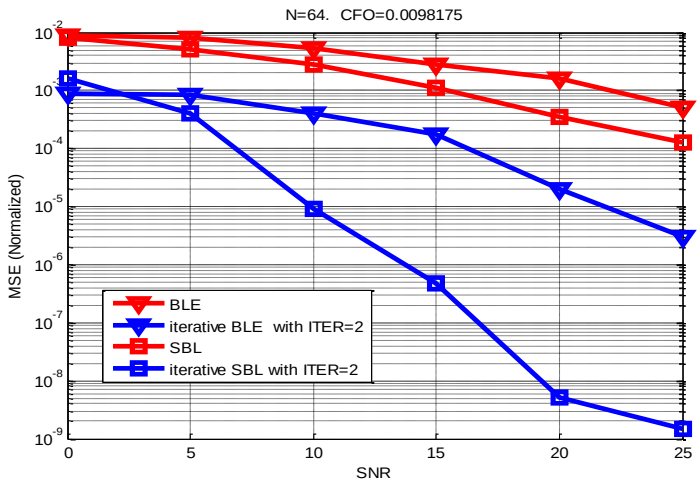

Figure 4. MSE versus SNR

In figure 4 we compare the iterative CFO estimation with no iterative algorithm, for both blind and semi blind in multipath channel with actual CFO$\varepsilon_0 = 0.1\omega$.

The accuracy of the Taylor Series approximation depends on the number of terms used in the summation and the actual value of the residual carrier offset.

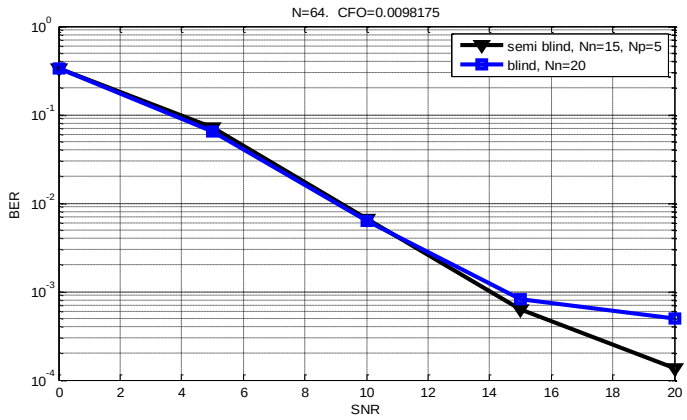

Figure 5. BER vs SNRS

We can see from figure 4 that the performance of the iterative method improved significantly with no error floor effect visible as the number of iterations increases, and that the semi blind method is better than the blind in the same conditions.

Figure 5 shows BER versus SNR. The semi-blind and blind CFO estimator curves are indistinguishable in the range 0 to 10 dB. But the semi blind method provides superior performance for high value of SNR (for SNR superior to 10 dB) as compared to the blind method.

VI. Conclusion

A low complexity semi-blind CFO estimation, based on pilot tones and virtual subcarriers is proposed for OFDM systems. The algorithm estimates the CFO by truncating Taylor's series to a few terms as in [1]. We show by simulation that the semi blind method outperforms the blind one in Gaussian and in multipath channels.

The derived methods are very suitable for real wireless environments since they require only one OFDM block for blind and semi-blind reliable estimation of CFO.

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